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On the Possibility of Inference to the Best Explanation

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Abstract Various proposals have suggested that an adequate explanatory theory should reduce the number or the cardinality of the set of logically independent claims that need be accepted in order to entail a body of data. A (and perhaps the only) well-formed proposal of this kind is William Kneale's: an explanatory theory should be finitely axiomatizable but it's set of logical consequences in the data language should not be finitely axiomatizable. Craig and Vaught showed that Kneale theories (almost) always exist for any recursively enumerable but not finitely axiomatizable set of data sentences in a first order language with identity. Kneale's criterion underdetermines explanation even given all possible data in the data language; gratuitous axioms may be "tacked on." Define a Kneale theory, T, to be logically minimal if it is deducible from every Kneale theory (in the vocabulary of T) that entails the same statements in the data language as does T. If they exist, minimal Kneale theories are candidates for best explanations: they are "bold" in a sense close to Popper's; some minimal Kneale theory is true if any Kneale theory is true; the minimal Kneale theory that is data equivalent to any given Kneale theory is unique; and no Kneale theory is more probable than some minimal Kneale theory. I show that under the Craig-Vaught conditions, no minimal Kneale theories exist.

Keywords Explanation · Finite axiomatizability · Minimal theories

1 Merging Kneale, Popper and Probability

Gilbert Harman [7] once offered that all explanation is inference to the best explanation. Presumably Harman meant inference to the best available explanation,

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not to the best possible explanation by some set of logical or probabilistic standards. Science and philosophy have longed for methods of the latter kind. What is wanted are objective criteria for assessing the goodness of possible explanations of a body of data, and for finding, if not *the* best explanation, at least one of the best explanations. Moreover, we would like a connection between best explanations and truth, at least a "pragmatic vindication" that would guarantee that, if any explanation of all possible data of a specified kind is true, then a best explanation is true. Nothing of the sort has emerged from discussions of Harman's idea, and commentaries have chiefly collected anecdotes [3, 13]. Methodological proposals from the last century suggest a coherent path to the ideal—and results in logic from the same period defeat it.

According to Karl Popper [14] it is best for theories to be bold, to be as falsifiable as possible. Suppose, with some of the philosophers of the last century, that all possible data of a definite kind can be formalized in a first-order language in a definite vocabulary. By "data" I mean to include both singular sentences and generalizations in this language, arbitrary first order sentences, which we may regard as statements of hypothetical empirical laws. Suppose that at least in limit of all observations of a given type the individual generalizations and singular sentences of this language represent a body of testable claims, either verifiable or falsifiable in the limit, or both.[1] In the spirit of Popper's standard it would seem better that a theory entail more such testable claims rather than fewer, but that is not quite right. More theoretical boldness requires more *independently* testable claims—at least logically independent. An ideally bold theory to account for some finite collection of data statements would therefore also entail an infinite collection of further testable claims about possible data, a collection that is not finitely axiomatizable in the language of the data. Of course, an infinity of logically independent testable consequences may very well not be all that there is to "boldness," but it would seem a feature that, other things equal, would contribute to boldness.

William Kneale [12] endorsed a similar requirement for a best scientific explanation::

> When we explain a given proposition we show that it follows logically from some other proposition or propositions. But this can scarcely be a complete account of the matter...An explanation must in some sense simplify what we have to accept. Now the explanation of laws by showing that they follow from other laws is a simplification of what we have to accept because it reduces the number of untransparent necessitations we need to assume...What we can achieve...is a reduction of the number of independent laws we need to assume for a complete description of nature.

Kneale went on to claim that an explanatory theory should be finitely axiomatizable but its set of "observable" consequences should not be: any finite set of axioms for a theory should entail all of its observational consequences, and further claims that are not in the observational vocabulary. That idea combines Popper's boldness criterion with an objective distinction in the cardinality of sets of

[1] For logically precise characterizations of these notions, see [9]. Existentially quantified sentences will therefore count as "testable" in my discussion, although not in Popper's.

"untransparent necessitations." It can be objected that theories have no such implications, that empirical results are only obtained by adding auxiliary hypotheses to them. This is mainly a matter of what is called a "theory." Celestial dynamics, for example, seems to illustrate Kneale's conception. The combination of Newtonian dynamics, the gravitational force law, and the claim that all forces between celestial bodies, considered as point particles, are gravitational, forms a finitely axiomatizable theory (or at least a finitely axiomatizable addition to a mathematical background), but has an infinity of consequences about how various numbers of bodies will move under sundry initial conditions. It is beside the point that this particular theory is false, and that we do not know how to calculate most of its implications analytically. It can be objected that some theories that are not finitely axiomatizable are nonetheless surveyable because an infinite collection of axioms are instances of a single axiom schema, for example the induction schema in Peano's postulates, or the continuity schema in first order Euclidean and Hyperbolic geometries. But Kneale was interested in reducing the cardinality of the set of assumptions, not only in surveyability or transparency. Finally, it can be objected that "observable" is notoriously vague. But in many cases in science a theory addresses a set of quantities with established criteria for their measurement, and for Kneale's proposal it is inessential whether they meet some philosophical criterion of "observability."

Adding any sentence to a finitely axiomatizable theory yields a finitely axiomatizable theory, a version of the logical fact that Mary Hesse [8] called the "tacking on" paradox. It implies that even in the infinite limit of all possible data of a specified kind, indeed even given an answer to every question that could be posed in the language of the data, Kneale's criterion does not determine a unique theory, let alone provide a principle for significantly reducing the number of candidates. Reichenbach [17] proposed to solve that underdetermination with a novel semantics: empirically equivalent theories are really synonymous; their empirical consequences are their truth conditions. Whatever the semantic disadvantages, that proposal comes with too high a logical price. On Reichenbach's semantics, entailment is not even recursively enumerable; no effective theory of proof is possible.[2]

Finite axiomatizability of a not-finitely axiomatizable collection of possible data is a criterion too weak. Since any finite collection of sentences is trivially logically

[2] To show that no effective proof theory is possible for Reichenbach's semantics it is sufficient to show that the consequence relation generated by the proposed truth conditions is not recursively enumerable (r.e.): Let L be a first-order language (with identity), V a proper sub-language of L such that there is at least one binary predicate which is in L but not in V. Let R be a relation on pairs of well-formed formulas of L such that for all sentences A, B in L, R(A, B) holds if and only if for every sentence S of V, S is provable from A only if S is provable from B in a standard first-order proof theory; this is Reichenbach's entailment relation.. Then R is not r.e. Proof: Let T be a consistent sentence of L which is undecidable and such that the consequences of T expressible in V alone are complete in V. A sentence Q of L is then refutable from T just if for some sentence A, R (A & ~A, T & Q) is true. Again, a sentence Q of L is irrefutable from T just if R (T & Q, T) is true. Assuming that R is r.e., it follows that both the set of sentences refutable from T and the set of all sentences irrefutable from T are r.e. Since each of these sets is the complement of the other, both are recursive. Hence the set of all sentences refutable from T is recursive so T is decidable, which is a contradiction. I thank Richard Grandy for this argument.

equivalent to a single sentence, finer numbering distinctions than Kneale's would seem to require some counting principle for the number of putative laws in a sentence, or at least some ordering of explanations by the content they postulate. How should we count the *number* of "untransparent necessitations" in a finitely axiomatizable theory? Citing the passage above from Kneale, Friedman [5] attempted a counting principle for sets of sentences in terms of "independent acceptability." His proposal had various technical difficulties that seem to have proved insuperable [10].

If Kneale's criterion is too weak to connect explanation with truth, it seems too strong to allow explanations to be more probable than all that they would explain. Van Fraassen [20] has noted that it is (probabilistically) incoherent to assign a higher probability to a theory than to the collection of testable consequences of the theory. The theory, after all, entails all sentences in the collection, and if probability respects entailment then the theory can be no more probable than what it entails. So a theory meeting Kneale's criterion can never be more probable than its set of observational consequences. So if the probability ordering of sets of propositions were the proper and entire rational basis for a preference about which sets of propositions to believe or to accept, there could be no probabilistic grounds for preferring a Kneale theory to its set of observational consequences.

The obvious and decisive response to this argument is that the principle that we should accept or believe only the maximally probable set of propositions, regardless of informativeness or other virtues, would lead to accepting only logical truths.

A preference for more probable propositions or sets of propositions can be combined with other criteria, for example by first partially ordering propositions or sets of propositions by those criteria, and then refining that ordering by probability. On grounds other than probability, one can prefer theories meeting Kneale's criterion to their respective sets of testable consequences, but among theories meeting Kneale's criterion, probabilities can respect the partial ordering of purely deductive entailment. If theory T meeting Kneale's criterion deductively entails theory Q also satisfying that criterion, then Q must be at least as probable as T. Thus, for any collection of observationally equivalent theories that meet Kneale's criterion, a logically weakest such theory—one that is logically entailed by every Kneale theory that has the same observational consequences—would be maximally probable. So we have a proposal for a constraint on inference to the best explanation. Given any finite body of evidence, there will be Kneale theories that entail that evidence, and of course (since they satisfy Kneale's criterion) also entail an infinity of other logically independent observational propositions. Among those Kneale theories, the proper subset of logically weakest Kneale theories that entail the finite data are to be preferred, because that is the set of most probable theories meeting Kneale's criterion.

The condition is actually stronger than this, since the logically weakest Kneale theory explanation of the (not finitely axiomatizable) set of its testable sentences is unique: if there were two logically inequivalent weakest finitely axiomatizable theories for the same (not finitely axiomatizable) set of testable sentences, neither could entail the other and their disjunction would be still weaker.

What is the connection with truth? There is no guarantee that true theories meet Kneale's. criterion, and so no guarantee that we could use observational data to find the truth even in the limit as the number of logically independent observational claims increases without bound.[3] But there appears to be a proper subclass of theories one of which must be true if any theory meeting Kneale's criterion is true: the class of logically (i.e., deductively) weakest theories meeting Kneale's criterion. Since some logically weakest theory meeting Kneale's criterion would be true if any theory meeting the criterion and having the same testable consequences were true, we have a connection with truth—not all that we would like, but some—and a kind of pragmatic vindication. Of course, a preference for logically weakest theories does not solve the problem of induction from finite evidence, since finite evidence might be entailed by many, mutually inconsistent, logically weakest theories meeting Kneale's criterion. Further, for various not finitely axiomatizable classes of observation statements, it is at least conceivable that there are no true theories satisfying Kneale's criterion: all Kneale theories entailing that class of observation sentences might be false.

Logically weakest theories meeting Kneale's criterion nonetheless appear to combine the following virtues.

1. Given an effective distinction in vocabulary between directly testable and not directly testable claims, the preference relation on theories is objective.
2. If any Kneale theory is true, a logically weakest Kneale theory must be true.
3. No Kneale theory can be more probable than an observationally equivalent, logically weakest Kneale theory.
4. With respect to logical independence of testable claims, Kneale theories are bold.

The desiderata are in some respects reconciled: Popper's boldness; Kneale's reduction of "non-transparent, contingent laws"; a preference for more probable theories; and, for any (not finitely axiomatizable) body of data, a unique logically weakest explanation that is true if any explanation of that data true. The proposal is that a logically weakest theory meeting Kneale's criterion is one of the best explanations of any set of its contingent, testable consequences. One might ask for more from a best explanation, but can one ask for this? Are there, then, any best explanations in the sense just described?

We can set aside one worry. Theories might be formulated in different languages, and the number of languages might be infinite. Then, there would be no weakest Kneale theory. There are ways round that. We can take theories to be equivalent if they have a common definitional extension that leaves the language of testable claims invariant.[4] Or, we can standardize the theoretical language. Any first order theory can be reformulated by definitional extensions from a theory with a single

[3] Even the truth or falsity of sentences in the observation language could not in general be decided in the limit. First order sentences may be undecidable in the limit from infinite sequences of variable free sentences. See H. Putnam [15]. The arguments that follow would, however, hold if each member of the set of observational sentences were required to be decidable in the limit, supposing such a collection not to be finitely axiomatizable.

[4] For example, the first order theories of real closed fields and first order Euclidean geometry with unit have a common definitional extension. See [4] and [6].

binary predicate [16, 19]. We may assume that all of the theories could in principle be formulated to extend the language of the testable sentences by the addition of claims formulated in terms of such a single extra predicate. But that does not show that best explanations exist.

The consequences of any best explanation, if it exists, will be a recursively enumerable collection of sentences. Consider any definite infinite, not finitely axiomatizable collection of potential data, and extensions of that collection by a finitely axiomatizable theory in extra predicates. The extension must be conservative, that is, it must entail, in the language of the data, all and only the sentences in the infinite collection and their consequences. Say a vocabulary for a first order language is finite if the set of predicate symbols, function symbols and constant symbols is finite. The questions are then:

1. *Under what conditions on a recursively enumerable but not finitely axiomatizable set O of first order sentences in finite vocabulary Lo does there exist a finitely axiomatizable conservative extension T of O in extra predicates?*
2. *Given a recursively enumerable set O of first-order sentences in finite vocabulary Lo having a finitely axiomatizable extension T in language Lt containing extra predicates not in Lo, such that T is a conservative extension (over Lo) of O,* under what conditions on *O does there exist a logically weakest such theory in Lt?*

The answer to question 1 is known for all but a special case. The next section reviews that answer and the structure of the theory construction in the proof, which is relevant to question 2. The final section answers question 2 as completely as the available answers to question 1.

2 The Existence of Theories Meeting Kneale's Criterion

Kleene [11] showed that any first order theory with only infinite models has a finitely axiomatizable conservative extension in extra predicates, and Craig and Vaught [1] strengthened this result to the following: any first order recursively enumerable set of sentences with at most a finite number of non-isomorphic finite models has such an extension. The proof contains a recipe for constructing such theories which will be useful in the next section.

Assume an expanded language L of a first order language Lo and a consistent, recursively enumerable set O of sentences in Lo. $Q^{(N)}$ is a finitely axiomatizable fragment of number theory capable of representing all recursive functions [19]. Let $\Delta_1 \ldots \Delta_n \ldots$ be a recursive sequence of terms in $Q^{(N)}$. Since the Godel codings V, F, respectively, of the vocabulary and well-formed formulas of Lo are recursive, and, by Craig's Theorem [2], the set of Godel codings of the axioms of O are recursive, there are formulas Θ_1, Θ_2. Θ_3. Θ_4, in $Q^{(N)}$ that represent those classes:

$$\begin{aligned}
&Q^{(N)}|{-}\Theta_1(\Delta_m) \text{ if } m \in F; Q^{(N)}|{-} \sim \Theta_1(\Delta_m) \text{ otherwise}\\
&Q^{(N)}|{-}\Theta_2(\Delta_m) \text{ if } m \in V; Q^{(N)}|{-} \sim \Theta_2(\Delta_m) \text{ otherwise}\\
&Q^{(N)}|{-}\Theta_3(\Delta_m) \text{ if } m \in Ax;; Q^{(N)}|{-} \sim \Theta_3(\Delta_m) \text{ otherwise}\\
&Q^{(N)}|{-}\Theta_4(\Delta_m, \Delta_n, \Delta_p) \text{ if } C(m,n) = p; Q^{(N)}|{-} \sim \Theta_4(\Delta_m, \Delta_n, , \Delta_p) \text{ otherwise,}
\end{aligned}$$

where "Cn" is the concatenation operation

The Craig-Vaught theory uses extra constant symbols, Po…Pp-1, Fm, Vb, Ax, As, and E,, the extra function symbols Cn, As, St, Vl, and the symbols of $Q^{(N)}$ (N, 0, +,),. The intended meaning of As(x) is "x is an assignment"; the meaning of Vl(x,u) = z is "the value of x at variable u is z"; and the meaning E(x, x', u, z) is "assignments x and x' have equal values at all variables except at most the variable u and the value of x' at u is z". Ax is the recursive predicate that holds of all and only the Godel numbers of the axioms of O represented by a formula as in Θ_3 above. The axioms of the Craig-Vaught theory are the universal closures of the following (which I quote, p. 296):

I The axioms of $Q^{(N)}$.

II .1 $\mathrm{Fm}(x) \leftrightarrow \theta_1 \wedge \mathrm{N}(x)$.

.2 $\mathrm{Vb}(x) \leftrightarrow \theta_2 \wedge \mathrm{N}(x)$.

.3 $\mathrm{Ax}(x) \leftrightarrow \theta_3 \wedge \mathrm{N}(x)$.

.4 $\mathrm{N}(x) \wedge \mathrm{N}(y) \wedge \mathrm{N}(z) \rightarrow [\mathrm{Cn}(x, y)=z \leftrightarrow \theta_4]$.

.5 $\mathrm{E}(x, x', u, z) \leftrightarrow \mathrm{As}(x) \wedge \mathrm{As}(x') \wedge \mathrm{Vb}(u) \wedge \mathrm{Vl}(x', u)=z$

$\wedge \bigwedge u'[\mathrm{Vb}(u') \wedge u' \neq u \rightarrow \mathrm{Vl}(x', u')=\mathrm{Vl}(x, u')]$.

III.1 $\bigvee x\, \mathrm{As}(x)$.

.2 $\mathrm{As}(x) \wedge \mathrm{Vb}(u) \rightarrow \bigvee x' \mathrm{E}(x, x', u, z)$.

Before stating the remaining axioms, we introduce inductively a new notation, by requiring that if $\tau_0, \ldots, \tau_n$ are terms of T_1, then $\tau_0{}^\frown \ldots {}^\frown\tau_n = \tau_0$ if $n = 0$, while $\tau_0{}^\frown \ldots {}^\frown\tau_n = \mathrm{Cn}(\tau_0{}^\frown \ldots {}^\frown\tau_m, \tau_{m+1})$ if $n = m+1$.

IV .1 $\mathrm{As}(x) \wedge \mathrm{Vb}(v_0) \wedge \ldots \wedge \mathrm{Vb}(v_{\rho_k-1}) \rightarrow [\mathrm{St}(x, \Delta_{\mathrm{P}_k}{}^\frown v_0{}^\frown \ldots {}^\frown v_{\rho_k-1})$

$\leftrightarrow \mathrm{P}_k(\mathrm{Vl}(x, v_0), \ldots, \mathrm{Vl}(x, v_{\rho_k-1})] \qquad (k = 0, \ldots, p)$.

.2 $\mathrm{As}(x) \wedge \mathrm{Fm}(w) \wedge \mathrm{Fm}(w') \rightarrow [\mathrm{St}(x, \Delta_/{}^\frown w^\frown w') \leftrightarrow \mathrm{St}(x, w)/\mathrm{St}(x, w')]$.

.3 $\mathrm{As}(x) \wedge \mathrm{Fm}(w) \wedge \mathrm{Vb}(u) \rightarrow \{\mathrm{St}(x, \Delta_\wedge{}^\frown u^\frown w) \leftrightarrow$

$\bigwedge z \bigwedge x'[\mathrm{E}(x, x', u, z) \rightarrow \mathrm{St}(x', w)]\}$.

V $\mathrm{As}(x) \wedge \mathrm{Ax}(w) \rightarrow \mathrm{St}(x, w)$.

In our context, the Craig-Vaught theory is simply an indirect, finitely axiomatizable way of asserting all of the axioms of an axiomatization of its set of testable or observational consequences. Focus on axiom 5.

3 On the Existence of Best Possible Explanations

The work of Kleene, Craig and Vaught proves that candidate Kneale theories generally exist, and our remaining question is whether there are logically weakest such theories. There is an easy negative result: the theory of infinity in the identity predicate has no logically weakest finitely axiomatizable extension. For suppose there were a sentence, W, which is the logically weakest sentence axiomatizing this theory. Then, any sentence S having only infinite models must entail W. And conversely, if S entails W, then it must have only infinite models, because W has only infinite models. So the set of sentences with only infinite models is exactly the set of sentences entailing W. But the set of sentences entailing W is recursively enumerable (by enumerating the proofs starting with S, we can find that it entails W, if in fact it does), and by a theorem of Vaught's [21], the set of sentences with only infinite models is not recursively enumerable.

The negative result extends to all of the theories meeting the Craig-Vaught condition, although the argument is different. Starting with any recursively axiomatizable theory, O, Axiom 5 of the Craig Vaught theory for O then says that for all assignments x, and for all values m of variable w, if Ax holds of m (i.e., if m is the Godel number of an axiom of O) then assignment x satisfies the axiom numbered by m. Call that theory $CV0_0$. Let O_i index a set of axioms for O in some sequence, and assume that no logical truth is in that set.[5] Let m_i be the Godel number of O_i. Let Ax_n be the formula $Ax(x) \& \sim x = m_1 \& \ldots \& \sim x = m_n$. Ax_n is a recursive predicate because $O \backslash \{O_1 \& ... \& O_n\}$ differs from O by a finite set. Let CVO_n be the theory obtained by replacing the occurrence of "Ax" in axiom 5 in CVO by "Ax_n" and adding the further axiom $O_1 \& ... \& O_n$. CVO_n logically entails O and is equivalent over the language of O to CVO_0. For $n>0$, CVO_n does not contain the axiom V: As(x) & Ax(u) –> St(x, u), but instead the axiom V_n: As(x) & Ax_n(u) –> St(x, u). Since every number that satisfies Ax_n also satisfies Ax, but not conversely, axiom V entails axiom V_n, but not conversely, and these axioms are each independent of the other four sets of Craig-Vaught axioms. So CVO_0 entails CVO_n for $n>0$, but not conversely. And in general, by an obvious induction, for all n there exists an m such that CVO_n entails CVO_{n+m}, but not conversely.

Every theory in the CVO_n sequence entails O, and therefore the intersection, $\cap_n Con(CVO_n)$, of the consequences of each theory in the sequence entails O. That intersection is axiomatized by the union of two sets of sentences: the axioms O_i of O constitute one set; the second set consists of the axioms of Craig and Vaught's groups I–IV, and the existential closures of the axiom variable of axioms V_n: $\exists w \forall x$ (As(x) & Ax_n(w) –>St(x,w)). The second set of axioms is satisfied by any model in which the Craig-Vaught axioms I–IV axioms are true, the existential closures of the V_n are true, and an infinite subset, F, of the axioms of O are false.[6] (The existential closures of the V_n will be vacuously true for all n indexing a conjunction of axioms in F because for all Godel numbers, w, of axioms O_i of O in F, the second term in the respective antecedents, Ax_n(w), in V_n , will not be satisfied by all assignments x.) Conversely, the O axioms have a model in which they are true but the Craig-Vaught axioms I–V are false. It follows that not all of the axioms of O can be deduced from the second set of axioms of $\cap_n Con(CVO_n)$, or from the second set and any finite subset of the axioms O_i of O. Therefore, $\cap_n Con(CVO_n)$ is not finitely axiomatizable. If there were a logically weakest Kneale theory, W, for O then W would also be

[5] The assumption is unnecessary but avoids technical complications later.

[6] Craig's theorem implies that every recursive set of first order sentences is recursively axiomatizable, but not every such set is recursively axiomatizable by axioms all of which are independent. So the set of true formulas of O in the model should be chosen so that they do not entail any O axiom that is in F, their infinite complement with respect to O.

entailed by each theory in the sequence, and hence would also be in $\cap_n \mathrm{Con}(\mathrm{CVO}_n)$. Hence $\cap_n \mathrm{Con}(\mathrm{CVO}_n)$ would be finitely axiomatizable. By contradiction, there is no logically weakest Kneale theory for O.

The argument leaves unsettled whether there is, somewhere, a recursively enumerable set of first order sentences that has a best explanation in the sense examined here—it will have to have an infinite number of non-isomorphic finite models. A reasonable conjecture is that no "best explanations" of this kind exist. If so, a demand for a logically weakest Kneale theory, or a most probable Kneale theory, can never be met: for any Kneale theory there is another logically weaker alternative that is at least as probable. I draw two morals: first, any objective theory of inference to the best explanation will need to appeal to principles other than, or in addition to, Kneale's criterion and maximal probability; and second, the beautiful literature on the model theory and proof theory of first order logic that flourished in the middle of the last century was, and perhaps still is, a marvelous but underused tool for philosophical inquiry.[7]

References

1. Craig, W., & Vaught, R. L. (1958). Finite axiomatizability using additional predicates. *The Journal of Symbolic Logic, 23*, 289–308.
2. Craig, W. (1953). On axiomatizability within a system. *The Journal of Symbolic Logic, 18*, 30–32.
3. Day, T., & Kincaid, H. (1994). Putting inference to explanation in its place. *Synthese, 98*, 271–295.
4. De Bouvere, K. (1965). Synonymous theories. In J. Addison, et al. (Eds.), *The theory of models*. Amsterdam: North-Holland.
5. Friedman, M. (1974). Explanation and scientific understanding. *The Journal of Philosophy, 71*, 5–19.
6. Glymour, C. (1970). Theoretical realism and theoretical equivalence. In *Proceedings of the biennial meeting of the Philosophy of Science Associatio*n (275–288).
7. Harman, G. (1965). The inference to the best explanation. *The Philosophical Review, 74*, 88–95.
8. Hesse, M. (1974) *The structure of scientific inference*. New York: Macmillan.
9. Kelly, K. (1996). *The logic of reliable inquiry*. New York: Oxford University Press.
10. Kitcher, P. (1976). Explanation, conjunction and unification. *The Journal of Philosophy, LXXIII*, 207–212.
11. Kleene, S. (1952). Finite axiomatizability of theories in the predicate calculus using additional predicates. *Memoirs of the American Mathematical Society, 10*, 27–68.
12. Kneale, W. (1949). *Probability and induction* (pp. 91–92). Oxford: Oxford University Press.
13. Lipton, P. (1991) *Inference to the best explanation.* New York: Routledge.
14. Popper, K. (1992). *The logic of scientific discovery*. New York: Routledge.
15. Putnam, H. (1965). Trial and error predicates and the solution to a problem of Mostowski. *Journal of Symbolic Logic, 20*, 49–57.
16. Quine, W. V. O. (1954). Reduction to a dyadic predicate. *The Journal of Symbolic Logic, 19*, 180–182.
17. Reichenbach, H. (1938). *Experience and prediction*. Chicago: University of Chicago Press.
18. Tarski, A. (1954). A general theorem concerning the reduction of primitive notions. *The Journal of Symbolic Logic, 19*, 158–159.
19. Tarski, A., Mostowski, A., & Robinson, R. M. (1953) *Undecidable theories*. Amsterdam: North Holland.
20. Van Fraassen, B. (1989) *Laws and symmetries*. New York: Oxford University Press.
21. Vaught, R. (1960). Sentences true in all constructive models. *The Journal of Symbolic Logic, 25*, 39–53.

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